TREE REPRESENTATIONS FOR BOREL FUNCTIONS

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Abstract

In 2009 Brian Semmes, in his PhD thesis, provided a characterization of Borel measurable functions from and into the Baire space, the space of infinite sequences of natural numbers with the product topology, using a reduction game called the Borel game, in which Player I constructs an infinite sequence of natural numbers and Player II constructs a tree hainvg a unique infinite branch. As in the other reduction games, Player II is required, in order to win, to guess correctly the image of the sequence played by Player I through a given function, which is common-knowledge. Around the same year, Alain Louveau wrote some (still unpublished) notes in which he provided a characterization of Baire class α functions (again from and into the Baire space), for all fixed $\alpha < \omega_1$ and, importantly, $\mathbf{\Sigma}^0_\lambda$-measurable functions for λ countable limit, using tree-representations instead of games. The interest in these problems was recently rekindled by the works of Nobrega, who proved in his PhD thesis a characterization of Baire class functions using Semmes' Borel game (yet missing the limit case), and many others like A. Andretta, T. Kihara, V. Gregoriades, V. Kiss, L. Motto Ros, R. Carroy and B. D. Miller. In this talk, I present Louveau's characterization, which both simplifies and extends Nobrega's work, by considering also the limit case. We also see that if we modify a bit the Borel game we end up characterizing functions having a G_δ graph, and that under AC there are functions for which the Borel game is undetermined. This last result opens questions regarding the consistency strength of the general determinacy of this game.

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